27. V. K. Saxena and S. C. Saxena, "Measurement of the thermal conductivity of argon using hot-wire type thermal diffusion columns," Chem. Phys. Lett., 2, No. 1, 44-46 (1968).
29. D. A. Timrot and A. S. Umanskii, "Investigation of the thermal conductivity of hydrogen and argon," Teplofiz. Vys. Temp., 4, No. 2, 289-293 (1966).
30. E. F. Smiley, The Measurement of the Thermal Conductivity of Gases at High Temperatures with a Shocktube; Experimental Results in Argon at Temperatures between 1000°K and 3000°K, Ph.D. Thesis, The Catholic University of America, Washington (1957).
31. M. R. Lauver, "Shock-tube thermal conductivity," Phys. Fluids, 7, No. 4, 611-612 (1964).
32. Yu. P. Zemlyanykh, "Experimental investigation of the thermal conductivity of gases at high temperatures in a shock tube," Author's Abstract of Candidate's Dissertation, Odessa Technological Institute of the Refrigeration Industry, Odessa (1972).
33. V. Kmoniček, J. Mastovský, and J. Malešak, "Tepelná vodivost dusiku a argonu za vysokých teplot," Str. Čas., Nos. 2-3, 348-358 (1968).
34. Yu. A. Polyakov and G. G. Spirin, "On the question of the investigation of the thermal conductivity of gases at high temperatures by the reflected-wave method," in: Thermophysical Properties of Gases [in Russian], Nauka, Moscow (1970), pp. 40-43.
35. R. G. Vines, "Measurement of the thermal conductivities of gases at high temperatures," J. Heat Transfer ASME, 82, No. 1, 48-52 (1960).
36. Yu. A. Gorshkov and A. S. Umanskii, "Investigation of the thermal conductivity of argon by a pulsed method," Teploenergetika, No. 10, 83-85 (1972).

EQUATIONS OF GENERALIZED THERMOELASTICITY OF A COSSERAT MEDIUM IN STRESSES

Vl. N. Smirnov

UDC 539.377

Thermoelasticity equations in stresses are derived in this paper for a Cosserat medium taking into account the finiteness of the heat propagation velocity. A theorem is proved on the uniqueness of the solution for one of the obtained systems of such equations.

The development of experimental investigation of the interaction between optical radiation and a substance evokes interest in a detailed study of the thermoelastic phenomena occurring in solids subjected to laser radiation. Such radiation requires taking account of the finiteness of the heat propagation velocity in connection with the quite rapid nature of the heat liberation process. Taking this circumstance into account requires insertion of an additional term in the Fourier heat conduction law, as is assumed in, e.g., [1, 2]. The polycrystalline or granular construction of many materials used in force optics evokes a requirement to involve a nonsymmetric Cosserat model in the analysis, which describes the behavior of such media more accurately under deformation [3]. The equations of isothermal nonsymmetric elasticity theory have been investigated in detail in [4-6]. The papers [7-9] and a section of the monograph [3] are devoted to the theory of nonsymmetric thermoelasticity without taking account of the finiteness of the heat propagation velocity. The equations of generalized thermoelasticity of a Cosserat continuum have been obtained in [10]. The system of equations in the displacement vector **u**, the small rotation vector **ω**, and the relative temperature deviation ϑ from the initial value Θ_0 has the form

$$
\begin{gathered}
(\mu+\alpha)\nabla^2\mathbf{u}+(\mu-\alpha+\lambda)\nabla\nabla\cdot\mathbf{u}+2\alpha\nabla\times\boldsymbol{\omega}+\mathbf{X}-\nu\Theta_0\nabla\vartheta=\rho\ddot{\mathbf{u}};\\
(\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\gamma-\varepsilon+\beta)\nabla\nabla\cdot\boldsymbol{\omega}+2\alpha\nabla\times\mathbf{u}-4\alpha\boldsymbol{\omega}+\mathbf{Y}=\mathbf{I}\cdot\ddot{\boldsymbol{\omega}};\\
k\nabla^2\vartheta-\tau_0 m\Theta_0\ddot{\vartheta}-m\Theta_0\dot{\vartheta}-\nu\tau_0\nabla\cdot\ddot{\mathbf{u}}-\nu\nabla\cdot\dot{\mathbf{u}}=-\Theta_0^{-1}w-\tau_0\Theta_0^{-1}\dot{w};\\
\vartheta=(\Theta-\Theta_0)\Theta_0^{-1}.
\end{gathered}
\quad (1)
$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 3, pp. 482-488, March, 1981. Original article submitted January 2, 1980.

Here μ, α, λ, ν, γ, ε, β, m are constants introduced in [3] that characterize the mechanical and thermophysical properties of the medium. Let us limit ourselves to examination of the case when

$$\mathbf{I} = I_0\mathbf{E}. \tag{2}$$

The system of equations (1) is a generalization of the classical Lamé and Fourier equations. As is known from the symmetric theory of elasticity, equations in the stress tensor are of interest in the solution of specific problems in many cases. To obtain such equations, a system of equations of motion in the force stresses T and the moment stresses M is necessary:

$$\nabla\cdot\mathbf{T} + \mathbf{X} = \rho\ddot{\mathbf{u}};\quad \nabla\cdot\mathbf{M} - 2\mathbf{a}^T + \mathbf{Y} = \mathbf{I}\cdot\ddot{\boldsymbol{\omega}}, \tag{3}$$

where a^T is a vector corresponding to the tensor T [11], and the system of governing equations is [3]

$$\mathbf{T} = 2\mu\boldsymbol{\gamma}^+ + 2\alpha\boldsymbol{\gamma}^- + (\lambda\boldsymbol{\gamma}\cdot\cdot\mathbf{E} - \nu\Theta_0\vartheta)\,\mathbf{E};$$
$$\mathbf{M} = 2\gamma\boldsymbol{\varkappa}^+ + 2\varepsilon\boldsymbol{\varkappa}^- + \beta(\boldsymbol{\varkappa}\cdot\cdot\mathbf{E})\,\mathbf{E};\quad s = \nu\boldsymbol{\gamma}\cdot\cdot\mathbf{E} + m\Theta_0\vartheta, \tag{4}$$

where

$$\boldsymbol{\gamma} = \nabla\mathbf{u} + \boldsymbol{\omega}\times\mathbf{E};\quad \boldsymbol{\varkappa} = \nabla\boldsymbol{\omega}. \tag{5}$$

The superscript plus denotes the symmetric part of the tensor, while the minus is for the antisymmetric part. It follows from the first equation in system (4) and the definition of the strain tensor (5) that

$$\nabla\cdot\mathbf{u} = \boldsymbol{\gamma}\cdot\cdot\mathbf{E} = \frac{1}{2\mu + 3\lambda}\,(\mathbf{T}\cdot\cdot\mathbf{E} + 3\nu\Theta_0\vartheta). \tag{6}$$

Using (6), the system of equations (4) is easily converted into the form

$$\boldsymbol{\gamma} = (2\mu)^{-1}\,\mathbf{T}^+ + (2\alpha)^{-1}\,\mathbf{T}^- + (2\mu + 3\lambda)^{-1}\,[\nu\Theta_0\vartheta - \lambda\,(2\mu)^{-1}\,\mathbf{T}\cdot\cdot\mathbf{E}]\,\mathbf{E};$$
$$\boldsymbol{\varkappa} = (2\gamma)^{-1}\,\mathbf{M}^+ + (2\varepsilon)^{-1}\,\mathbf{M}^{-1} - \beta\,[2\gamma\,(2\gamma + 3\beta)]^{-1}\,(\mathbf{M}\cdot\cdot\mathbf{E})\,\mathbf{E}; \tag{7}$$
$$s = \nu\,(2\mu + 3\lambda)^{-1}\,\mathbf{T}\cdot\cdot\mathbf{E} + [m + 3\nu^2\,(2\mu + 3\lambda)^{-1}]\,\Theta_0\vartheta.$$

If the following integrals are considered

$$\boldsymbol{\omega}(C) - \boldsymbol{\omega}(C_0) = \int_{C_0}^{C} d\boldsymbol{\omega} = \int_{C_0}^{C} \boldsymbol{\varkappa}^*\cdot d\mathbf{r};$$
$$\mathbf{u}(C) - \mathbf{u}(C_0) = \int_{C_0}^{C} d\mathbf{u} = \int_{C_0}^{C} (\boldsymbol{\gamma} - \boldsymbol{\omega}\times\mathbf{E})^*\cdot d\mathbf{r}, \tag{8}$$

then by requiring independence of the integrals from the form of the path of integration, we obtain the compatibility equations

$$\nabla\times\boldsymbol{\varkappa} = 0;\quad \nabla\times\boldsymbol{\gamma} - \boldsymbol{\varkappa}^* + (\mathbf{E}\cdot\cdot\boldsymbol{\varkappa})\,\mathbf{E} = 0. \tag{9}$$

The asterisk denotes the tensor that is the transpose of that given. The equations (9) were obtained in [5] by a somewhat different method. Using the governing equations (7), we easily obtain the following system of equations from the compatibility conditions (9):

$$\varepsilon\nabla\times\mathbf{M}^+ + \gamma\nabla\times\mathbf{M}^- - \beta\varepsilon\,(2\gamma + 3\beta)^{-1}\,\nabla\times(\mathbf{E}\,(\mathbf{M}\cdot\cdot\mathbf{E})) = 0;$$
$$(2\mu\alpha)^{-1}\,[\alpha\nabla\times\mathbf{T}^+ + \mu\nabla\times\mathbf{T}^- - \alpha\lambda\,(2\mu + 3\lambda)^{-1}\,\nabla\times(\mathbf{E}\,(\mathbf{T}\cdot\cdot\mathbf{E})) +$$
$$+ \nu\Theta_0\,(2\mu + 3\lambda)^{-1}\,\nabla\times\vartheta\mathbf{E} - (2\gamma\varepsilon)^{-1}\,[\varepsilon\mathbf{M}^+ - \gamma\mathbf{M}^- + \varepsilon\,(\beta - 2\gamma)\,(2\gamma + 3\beta)^{-1}\,(\mathbf{M}\cdot\cdot\mathbf{E})\,\mathbf{E}] = 0. \tag{10}$$

Moreover, the equations in the tensor components of the force and moment stresses can be obtained analogously to how the Beltrami–Mitchell equations were obtained for a symmetric medium in [12]. By using the "gradient" operation in the first two equations of the system (1), using the definition of the strain and bending-torsion tensors (5), and also the governing equations (7), we obtain the following system:

$$\left\{(\gamma+\varepsilon)\nabla^2 - I_0\frac{\partial^2}{\partial t^2}\right\}\left\{\frac{1}{2\gamma}\mathbf{M}^+ + \frac{1}{2\varepsilon}\mathbf{M}^- - \frac{\beta(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)}\right\} - \frac{2\alpha(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma+3\beta} + \nabla\mathbf{Y} + (\gamma-\varepsilon+\beta)\nabla\nabla\cdot\left\{\frac{1}{2\gamma}\mathbf{M}^+ - \frac{1}{2\varepsilon}\mathbf{M}^- -\right.$$

$$\left. - \frac{\beta(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)}\right\} - 2\alpha\left\{\frac{1}{2\gamma}\mathbf{M}^+ + \frac{1}{2\varepsilon}\mathbf{M}^- - \frac{\beta(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)}\right\} +$$

$$+ 2\alpha\left\{\nabla\times\left\{\frac{1}{2\mu}\mathbf{T}^+ + \frac{1}{2\alpha}\mathbf{T}^- + \frac{1}{2\mu+3\lambda}\left(\nu\Theta_0\vartheta - \frac{\lambda}{2\mu}\mathbf{T}\cdot\cdot\mathbf{E}\right)\mathbf{E}\right\}\right\}^* = 0; \tag{11}$$

$$\left\{(\mu+\alpha)\nabla^2 - \rho\frac{\partial^2}{\partial t^2}\right\}\left\{\frac{1}{2\mu}\mathbf{T}^+ + \frac{1}{2\alpha}\mathbf{T}^- - \frac{1}{2\mu+3\lambda}\left(\nu\Theta_0\vartheta - \frac{\lambda}{2\mu}\mathbf{T}\cdot\cdot\mathbf{E}\right)\mathbf{E}\right\} + \nabla\mathbf{X} - \nu\Theta_0\nabla\nabla\vartheta + (\mu-\alpha+\lambda)\nabla\nabla\cdot\left\{\frac{1}{2\mu}\mathbf{T}^+ -\right.$$

$$\left. - \frac{1}{2\alpha}\mathbf{T}^- + \frac{1}{2\mu+3\lambda}\left(\nu\Theta_0\vartheta - \frac{\lambda}{2\mu}\mathbf{T}\cdot\cdot\mathbf{E}\right)\mathbf{E}\right\} + \frac{\rho}{I_0}(\nabla\cdot\mathbf{M} - 2\mathbf{a}^T + \mathbf{Y})\times\mathbf{E} +$$

$$+ (\mu+\alpha+\lambda)\left\{\nabla\cdot\left\{\frac{1}{2\gamma}\mathbf{M}^+ - \frac{1}{2\varepsilon}\mathbf{M}^- - \frac{\beta(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)}\right\}\right\}^* -$$

$$- (\mu+\alpha)\left\{\nabla\cdot\left\{\frac{1}{2\gamma}\mathbf{M}^+ + \frac{1}{2\varepsilon}\mathbf{M}^- - \frac{\beta(\mathbf{M}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)}\right\}\right\}\times\mathbf{E} = 0.$$

It should be noted that other equations in T and M can also be obtained. By applying the "gradient" operation to (3), as is done in [13] in the examination of a symmetric medium, and using (5) and (7), we convert the system to the form

$$\frac{\rho}{2\mu}\ddot{\mathbf{T}}^+ + \frac{\rho}{2\alpha}\ddot{\mathbf{T}}^- - \nabla\nabla\cdot\mathbf{T} + \frac{2\rho}{I_0}\mathbf{T}^- - \frac{\rho\lambda(\ddot{\mathbf{T}}\cdot\cdot\mathbf{E})\mathbf{E}}{2\mu(2\mu+3\lambda)} - \frac{\rho}{I_0}(\nabla\cdot\mathbf{M})\times\mathbf{E} + \frac{\rho\nu\Theta_0\ddot{\vartheta}\mathbf{E}}{2\mu+3\lambda} - \nabla\mathbf{X} - \frac{\rho}{I_0}\mathbf{Y}\times\mathbf{E} = 0; \tag{12}$$

$$\frac{I_0}{2\gamma}\ddot{\mathbf{M}}^+ + \frac{I_0}{2\varepsilon}\ddot{\mathbf{M}}^- - \nabla\nabla\cdot\mathbf{M} - \frac{\beta I_0(\ddot{\mathbf{M}}\cdot\cdot\mathbf{E})\mathbf{E}}{2\gamma(2\gamma+3\beta)} + 2\nabla\mathbf{a}^T + \nabla\mathbf{Y} = 0.$$

It is perfectly evident that any of the systems (10), (11), and (12) should be supplemented by the heat conduction equation. Using (6), the heat conduction equation is easily transformed into the form

$$k\nabla^2\vartheta - \Theta_0\left(m + \frac{3\nu^2}{2\mu+3\lambda}\right)(\tau_0\ddot{\vartheta} + \dot{\vartheta}) - \frac{\nu}{2\mu+3\lambda}(\tau_0\ddot{\mathbf{T}}\cdot\cdot\mathbf{E} + \dot{\mathbf{T}}\cdot\cdot\mathbf{E}) = -\frac{w}{\Theta_0} - \frac{\tau_0\dot{w}}{\Theta_0}. \tag{13}$$

Attention should be turned to the circumstance that in the consideration of a symmetric medium the systems (10) and (11) go over into the Beltrami–Mitchell equations in the limit case, which, as is known, should be supplemented by equations of motion or equilibrium for solution. As regards (12), which are generalized Ignaczak equations, it can then be expected that the solution of this system supplemented by the heat conduction equation turns out to be unique.

Let the system of equations (12) and (13) have two different solutions $\mathbf{T}_1$, $\mathbf{M}_1$, ϑ_1 and $\mathbf{T}_2$, $\mathbf{M}_2$, ϑ_2 that satisfy boundary and initial conditions of the form

$$\begin{aligned} &\mathbf{n}\cdot\mathbf{T}|_\Phi = \mathbf{F}(t,\Phi); \quad \mathbf{n}\cdot\mathbf{M}|_\Phi = \mathbf{G}(t,\Phi); \quad \vartheta|_\Phi = H(t,\Phi);\\ &\mathbf{T}(K)|_{t=0} = \mathbf{f}(K); \quad \mathbf{M}(K)|_{t=0} = \mathbf{g}(K); \quad \vartheta(K)|_{t=0} = l(K);\\ &\dot{\mathbf{T}}(K)|_{t=0} = \mathbf{h}(K); \quad \dot{\mathbf{M}}(K)|_{t=0} = \mathbf{j}(K); \quad \dot{\vartheta}(K)|_{t=0} = r(K);\\ &(\cdot)\, K\in V. \end{aligned} \tag{14}$$

In this case $\mathbf{T}_0 = \mathbf{T}_1 - \mathbf{T}_2$; $\mathbf{M}_0 = \mathbf{M}_1 - \mathbf{M}_2$; $\vartheta_0 = \vartheta_1 - \vartheta_2$ satisfy the homogeneous equations (12) and (13) for homogeneous boundary and initial conditions. We integrate the homogeneous equations (12) with respect to time in the interval (0, t). If the initial conditions are taken into account and the tensors S and N, analogous to the Biot vector in the theory of heat conduction, are introduced

$$\dot{\mathbf{S}} = \mathbf{T}; \quad \dot{\mathbf{N}} = \mathbf{M}, \tag{15}$$

then we obtain as a result of the integration

$$\frac{\rho}{2\mu}\dot{\mathbf{T}}_0^+ + \frac{\rho}{2\alpha}\dot{\mathbf{T}}_0^- - \nabla\nabla\cdot\mathbf{S}_0 + \frac{2\rho}{I_0}\mathbf{S}_0^- - \frac{\rho\lambda(\dot{\mathbf{T}}_0\cdot\cdot\mathbf{E})\mathbf{E}}{2\mu(2\mu+3\lambda)} - \frac{\rho}{I_0}(\nabla\cdot\mathbf{N}_0)\,\mathbf{E} + \frac{\rho\nu\Theta_0\dot{\vartheta}_0\mathbf{E}}{2\mu+3\lambda} = 0;$$

$$\frac{I_0}{2\gamma}\,\dot{\mathbf{M}}_0^+ + \frac{I_0}{2\varepsilon}\,\dot{\mathbf{M}}_0^- - \nabla\nabla\cdot\mathbf{N}_0 - \frac{\beta I_0(\dot{\mathbf{M}}_0\cdot\cdot\,\mathbf{E})\,\mathbf{E}}{2\gamma(2\gamma+3\beta)} + 2\mathbf{a}_0^S = 0. \tag{16}$$

Let us combine the result of a convolution of the first of the tensor equations (16) with the tensor $\mathbf{T}_0$ and the second equation with the tensor $\rho\mathbf{I}_0^{-1}\mathbf{M}_0$. We combine the obtained expression with (13) multiplied by $-\rho\Theta_0\vartheta_0$, and we integrate over the volume of the body V and in time in the interval (0, t). Using the Ostrogradskii–Gauss theorem and taking account of the homogeneous boundary conditions, we obtain the following equality:

$$J_1 + J_2 + J_3 + J_4 + \int\limits_V \left\{\frac{\rho}{4\alpha}\,\mathbf{T}_0^-\cdot\cdot\,\mathbf{T}_0^- + \frac{\rho}{4\varepsilon}\,\mathbf{M}_0^-\cdot\cdot\,\mathbf{M}_0^- + \frac{1}{2}\,(\nabla\cdot\mathbf{S}_0)^2\right\} dV = 0, \tag{17}$$

where we have introduced the notation

$$J_1 = \frac{\rho}{4\gamma}\int\limits_V \left\{\mathbf{M}_0^+\cdot\cdot\,\mathbf{M}_0^+ - \frac{\beta}{2\gamma+3\beta}\,(\mathbf{M}_0\cdot\cdot\,\mathbf{E})^2\right\} dV;$$

$$J_2 = \int\limits_V \left\{\frac{\rho}{4\mu}\,\mathbf{T}_0^+\cdot\cdot\,\mathbf{T}_0^+ + \frac{\rho\nu\Theta_0\vartheta_0(\mathbf{E}\cdot\cdot\,\mathbf{T}_0)}{2\mu+3\lambda} - \frac{\rho\lambda(\mathbf{T}_0\cdot\cdot\,\mathbf{E})^2}{4\mu(2\mu+3\lambda)} + \frac{\rho\Theta_0^2}{2}\left(m + \frac{3\nu^2}{2\mu+3\lambda}\right)\vartheta_0^2\right\} dV; \tag{18}$$

$$J_3 = \rho\Theta_0\int\limits_0^t\int\limits_V \left\{k(\nabla\vartheta_0)^2 + \frac{\tau_0\nu\vartheta_0}{2\mu+3\lambda}\,(\ddot{\mathbf{T}}\cdot\cdot\,\mathbf{E}) + \tau_0\Theta_0\left(m + \frac{3\nu^2}{2\mu+3\lambda}\right)\ddot{\vartheta}_0\vartheta_0\right\} dVdt;$$

$$J_4 = \frac{\rho}{I_0}\int\limits_0^t\int\limits_V \left\{2\mathbf{M}_0\cdot\cdot\,\nabla\mathbf{a}_0^S - \mathbf{T}_0\cdot\cdot\,((\nabla\cdot\mathbf{N}_0)\times\mathbf{E}) + \frac{1}{2}\,\frac{\partial}{\partial t}\,(\nabla\cdot\mathbf{N}_0)^2 + \frac{\partial}{\partial t}\,(\mathbf{S}_0^-\cdot\cdot\,\mathbf{S}_0^-)\right\} dVdt.$$

Let us consider the integrals J_1, J_2, J_3, J_4. It is easy to show that

$$J_1 = \frac{\rho}{4\gamma}\int\limits_V \Big\{2\,[(M_0^+)_{12}^2 + (M_0^+)_{13}^2 + (M_0^+)_{23}^2] + 2\gamma(2\gamma+3\beta)^{-1}\,[(M_0)_{11}^2 +$$
$$+ (M_0)_{22}^2 + (M_0)_{33}^2] + \beta(2\gamma+3\beta)^{-1}\,\{[(M_0)_{11} - (M_0)_{22}]^2 + [(M_0)_{22} - (M_0)_{33}]^2 + [(M_0)_{33} - (M_0)_{11}]^2\}\Big\}\, dV. \tag{19}$$

As a result of simple calculations, the integral J_2 can be converted to the form

$$J_2 = \frac{\rho}{2}\int\limits_V \Big\{m\Theta_0^2\vartheta_0^2 + \frac{1}{\mu}\,[(T_0^+)_{12}^2 + (T_0^+)_{13}^2 + (T_0^+)_{23}^2] +$$
$$+ \lambda\,[2\mu(2\mu+3\lambda)]^{-1}\,\{[(T_0)_{11} - (T_0)_{22}]^2 + [(T_0)_{22} - (T_0)_{33}]^2 +$$
$$+ [(T_0)_{33} - (T_0)_{11}]^2\} + (2\mu+3\lambda)^{-1}\,\{[(T_0)_{11} + \nu\Theta_0\vartheta_0]^2 + [(T_0)_{22} + \nu\Theta_0\vartheta_0]^2 + [(T_0)_{33} + \nu\Theta_0\vartheta_0]^2\}\Big\}\, dV. \tag{20}$$

Using the third of the governing equations (7) and the Ostrogradskii–Gauss theorem, taking account of the boundary conditions, the entropy balance equation

$$\Theta_0(1+\vartheta)\,\dot{s} = -\nabla\cdot\mathbf{q} + w \tag{21}$$

and the generalized Fourier law [2]

$$\tau_0\dot{\mathbf{q}} + \mathbf{q} = -k\Theta_0\nabla\vartheta, \tag{22}$$

we convert the integral J_3. Since equations of linear thermoelasticity are considered in executing the transformations, only terms of identical order should be conserved. Consequently, we obtain

$$J_3 = -\rho\int\limits_0^t\int\limits_V \mathbf{q}_0\cdot\nabla\vartheta_0\,dVdt. \tag{23}$$

Let us examine the integral J_4. If the vector

$$c = \nabla \cdot N_0 \tag{24}$$

is introduced into the consideration, then it can be shown as a result of simple manipulation that

$$J_4 = \frac{\rho}{2I_0} \int_V \{[c_1 + (S_0)_{23} - (S_0)_{32}]^2 + [c_2 + (S_0)_{31} - (S_0)_{13}]^2 + [c_3 + (S_0)_{12} - (S_0)_{21}]^2\}\, dV. \tag{25}$$

As is known from thermodynamics [14],

$$-\frac{q \cdot \nabla\vartheta}{(1+\vartheta)^2\,\Theta_0} \geqslant 0, \tag{26}$$

hence, for ρ, I_0, α, ε, γ, β, μ, λ, $m > 0$ it follows from (17) that the integral of the sum of the quadratic terms is a nonpositive quantity. This is possible only if all the integrands vanish. Therefore,

$$T_0 = 0; \quad M_0 = 0; \quad \vartheta_0 = 0. \tag{27}$$

The system of equations (12) and (13) hence has a unique solution. It should be noted that in the limit case the system of equations (12) and (13) goes over into the system of equations of generalized thermoelasticity in stresses for a symmetric medium

$$\begin{gathered} \frac{\rho \ddot{T}}{2\mu} + \frac{\rho E}{2\mu + 3\lambda}\left(\frac{\lambda}{2\mu}\, T \cdot\cdot E - \nu\Theta_0\vartheta\right) = \operatorname{def} \nabla \cdot T + \operatorname{def} X; \\ k\nabla^2\vartheta - \Theta_0\left(m + \frac{3\nu^2}{2\mu + 3\lambda}\right)(\tau_0\ddot{\vartheta} + \dot{\vartheta}) - \frac{\nu}{2\mu + 3\lambda}(\tau_0\ddot{T} \cdot\cdot E + \dot{T} \cdot\cdot E) = -\frac{w}{\Theta_0} - \frac{\tau_0\dot{w}}{\Theta_0}, \end{gathered} \tag{28}$$

where

$$\operatorname{def} a = \frac{1}{2}(\nabla a + (\nabla a)^*), \tag{29}$$

and the proof presented for the uniqueness of the solution goes, for $\vartheta = 0$, $\nu = 0$, over into a proof of the uniqueness theorem for the solution of the Ignaczak equation of elastokinetics for a symmetric medium. In contrast to the proof presented in [3], nowhere are equations used that contain kinematic characteristics.

NOTATION

u, displacement vector; ω, small rotation vector; Θ, absolute temperature; Θ_0, initial temperature of the medium; ϑ, relative deviation of the temperature from the initial value; μ, α, λ, ν, ε, γ, β, m, constants characterizing the mechanical or thermophysical properties of the medium; ρ, density; **I**, dynamic characteristic of the medium reaction during rotation; k, heat conduction coefficient; τ_0, a constant characterizing the velocity of heat propagation; **X**, external volume force vector; **Y**, external volume moment vector; w, density of the heat liberation sources distributed in the medium; **E**, unit tensor; **T**, force stress tensor; **M**, moment stress tensor; γ, nonsymmetric strain tensor; $\varkappa$, bending-torsion tensor; s, entropy referred to unit volume; V, volume occupied by the body; Φ, surface bounding the body; $(T)_{ki}$, $(M)_{ki}$, components of the tensors **T** and **M**; q, thermal flux vector.

LITERATURE CITED

1. P. Vernotte, "Les paradoxes de la théorie continue l'équation de la chaleur," C. R. Acad. Sci., 246, No. 22, 3154-3155 (1958).
2. A. V. Lykov, "Application of thermodynamics of irreversible processes methods to heat and mass transfer investigations," Inzh.-Fiz. Zh., 9, No. 3, 287-304 (1965).
3. W. Nowacki, Thermoelasticity, Pergamon Press (1963).
4. V. A. Pal'mov, "Fundamental equations of nonsymmetric elasticity," Prikl. Mat. Mekh., 28, No. 3, 401-408 (1964).
5. N. Sandru, "On some problems of asymmetric elasticity," Int. J. Eng. Sci., 4, No. 1, 81-94 (1966).
6. R. D. Mindlin, "Microstructure in linear elasticity," Arch. Rat. Mech. Anal., 14, No. 1, 51-78 (1964).

7. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 2, 97-106 (1966).
8. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 3, 203-212 (1966).
9. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 8, 505-513 (1966).
10. Vl. N. Smirnov, "Equations of generalized thermoelasticity of a Cosserat medium," Inzh.-Fiz. Zh., 39, No. 4, 716-723 (1980).
11. A. I. Lur'e, Theory of Elasticity [in Russian], Nauka, Moscow (1970).
12. J. Ignaczak, "Direct determination of stresses from the stress equation of motion in elasticity," Arch. Mech. Stoss., 11, No. 5, 671-678 (1959).
13. J. Ignaczak, "A completeness problem for stress equation of motion in linear elasticity theory," Arch. Mech. Stoss., 15, No. 2, 225-234 (1963).
14. S. De Groot and P. Mazur, Non-Equilibrium Thermodynamics, American Elsevier (1962).

EXCITATION OF A TEMPERATURE WAVE BY A RECTANGULAR THERMAL SURFACE PULSE

A. G. Goloveiko and V. I. Martynikhina

UDC 537.52:536.3

We calculate the temperature field in a metal under the action of a rectangular thermal surface impulse with a fixed total energy and varying duration. It is shown that for a given duration of this impulse, conditions are created which ensure a maximal shift of the melting isotherm toward the interior of the metal.

In the present work we consider a thermophysical process in a metal in regimes when the following condition holds for the thermal pulse applied at the boundary:

$$W = Ft = \text{const}, \tag{1}$$

where F is the thermal flux density, which is constant during the time of its action t. The condition W = constant can be realized in various ways: from a short pulse of a high-density thermal flux, to an extended pulse for a low density of the thermal flux. Condition (1) essentially describes a multitude of pulses which differ by parameters F and t but have the same parameter W.

An analysis shows that the action of thermal pulses which differ in parameters F and t but have the same parameter W has appreciably different results on the metal.

For a long pulse duration the high thermal conductivity characteristic for metals ensures the transfer of the heat flux far into the metal. Therefore, the long pulse excites a deep but weak heating of the metal whose temperature field is extended over a large region. Towards the end of the pulse, the melting isotherm remains near the surface of the metal because of the weak heating. For short pulses of the same energy W, on the other hand, the metal is heated to large temperatures, and the temperature field is concentrated near the surface of the metal. In this case, the melting isotherm towards the end of the pulse also remains near the surface of the metal but for a different reason, because of the spatial concentration of the temperature field.

Clearly, in the intermediate conditions between long and short duration at a given energy W, the melting isotherm will be displaced by the largest amount. The aim of the present work is to substantiate this assertion quantitatively because of its importance in the analysis of the appropriate scientific and applied problems.

In the solution of the problem formulated above we shall limit ourselves to the analysis of a one-dimensional thermophysical process, and neglect the phase transformations. The process will be approximated by the problem of excitation of a temperature field (or a temperature wave) by a rectangular thermal pulse:

Belorussian Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 3, pp. 489-494, March, 1981. Original article submitted February 21, 1980.

